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NASA TECHNICAL MEMORANDUM

NASA TM X-64930

(NASA-TM-X-64930) STEERING LAW FOR PARALLEL
MOUNTED DOUBLE-GIMBALED CONTROL MOMENT GYROSCOPES
(NASA) 34 p PC \$3.75 CSCL 14B

N75-25129

Unclass

63/35 24156

STEERING LAW FOR PARALLEL MOUNTED DOUBLE-GIMBALED CONTROL MOMENT GYROSCOPES

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February 1975



NASA

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1. REPORT NO. NASA TM X-64930		2. GOVERNMENT ACCESSION NO.		3. RECIPIENT'S CATALOG NO.	
4. TITLE AND SUBTITLE Steering Law for Parallel Mounted Double-Gimbaled Control Moment Gyros				5. REPORT DATE February 1975	
				6. PERFORMING ORGANIZATION CODE	
7. AUTHOR(S) H. F. Kennel				8. PERFORMING ORGANIZATION REPORT #	
9. PERFORMING ORGANIZATION NAME AND ADDRESS George C. Marshall Space Flight Center Marshall Space Flight Center, Alabama 35812				10. WORK UNIT NO.	
				11. CONTRACT OR GRANT NO.	
				13. TYPE OF REPORT & PERIOD COVERED Technical Memorandum	
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D. C. 20546				14. SPONSORING AGENCY CODE	
15. SUPPLEMENTARY NOTES Prepared by Systems Dynamics Laboratory, Science and Engineering					
16. ABSTRACT <p>Mounting of double-gimbal control moment gyros (DG CMG's) of unlimited gimbal angle freedom with all their outer gimbal axes parallel allows drastic simplification in the CMG control law development in the redundancy management and failure accommodation and in the mounting hardware. The advantages of the parallel mounting for the CMG control law development are such that a law could be developed which is applicable to any number of DG CMG's. Parallel mounting of the DG CMG's in conjunction with the control law can therefore be considered a "CMG kit" suitable for many missions of differing momentum requirements. It also means that increasing momentum demands during the design phase of a space vehicle can be easily met by the addition of one or more CMG's of the original momentum capacity rather than a redesign to a larger momentum capacity. Another advantage of the parallel mounting is that the failure of any CMG can be treated like any other, i. e., only one failure mode is possible. The CMG steering law distributes the CMG momentum vectors such that all inner gimbal angles are equal which reduces the rate requirements on the outer gimbal axes. The steering law also maximizes the minimum angle between any two of the outer gimbals which ensures proper spacing of all the outer gimbals.</p>					
17. KEY WORDS Space vehicles Attitude control Pointing control Momentum exchange Control moment gyros			18. DISTRIBUTION STATEMENT Unclassified - Unlimited <i>H. F. Kennel</i>		
19. SECURITY CLASSIF. (of this report) Unclassified		20. SECURITY CLASSIF. (of this page) Unclassified		21. NO. OF PAGES 34	
				22. PRICE NTIS	

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DEFINITION OF SYMBOLS

Symbol	Definition
$[A]$	$3 \times n$ torque matrix (for inner rates)
a_{ij}	components of $[A]$; $i = 1, 2, 3$; $j = 1, 2, \dots, n$; $N \cdot m \cdot s$
$[B]$	$3 \times n$ torque matrix (for outer rates), $N \cdot m \cdot s$
b_{ij}	components of $[B]$; $i = 1, 2, 3$; $j = 1, 2, \dots, n$; $N \cdot m \cdot s$
c	cos (before Greek symbol)
D	divisor for rate limiting
E	$n \times n$ identity matrix
H_i	angular momentum magnitude of the i -th CMG; $i = 1, 2, \dots, n$; $N \cdot m \cdot s$
\underline{H}_G	total angular momentum of the CMG system; $N \cdot m \cdot s$
H_{G1}, H_{G2}, H_{G3}	components of \underline{H}_G ; $N \cdot m \cdot s$
$\dot{\underline{H}}_{GC}$	commanded CMG system momentum change; $N \cdot m$
$\dot{H}_{GC1}, \dot{H}_{GC2}, \dot{H}_{GC3}$	components of $\dot{\underline{H}}_{GC}$; $N \cdot m$
$\dot{\underline{H}}_G$	Actual CMG system momentum change; $N \cdot m$
$\dot{H}_{G1}, \dot{H}_{G2}, \dot{H}_{G3}$	components of $\dot{\underline{H}}_G$; $N \cdot m$
H_N	nominal CMG angular momentum; $N \cdot m \cdot s$
i	index
j	index
k_i	index of decreasing β -angle; $i = 1, 2, \dots, n$
K_A	distribution gain ($1/K_A$ is the time constant); $1/s$

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
m	number of failed CMG's (in flow charts)
n	number of DG CMCs
s	sin (before Greek symbol)
τ_C	control torque command
T_{C1}, T_{C2}, T_{C3}	components of \underline{T}_C ; $N \cdot m$
T_{CM2}, T_{CM3}	modified torque commands; $N \cdot m$
\underline{T}_G	total CMG torque on the vehicle; $N \cdot m$
T_{G1}, T_{G2}, T_{G3}	components of \underline{T}_G ; $N \cdot m$
T_{LIM}	maximum CMG gimbal torquer torque; $N \cdot m$
T_{1i}, T_{2i}, T_{3i}	torque components in the W-coordinate system; $i = 1, 2, 3 \dots n, N \cdot m$
V_1, V_2, V_3	vehicle geometric coordinate system axes
W_1, W_2, W_3	CMG coordinate system axes
α	average inner gimbal angle; rad
α_i, β_i	inner and outer gimbal angle; $i = 1, 2, \dots, n$; rad
α_{Ri}, β_{Ri}	inner and outer gimbal reference angle; $i = 1, 2, \dots, n$; rad
$\dot{\alpha}, \dot{\beta}$	inner and outer rate vector; rad/s
$\dot{\alpha}_i, \dot{\beta}_i$	inner and outer rates; $i = 1, 2, \dots, n$; rad/s
$\dot{\alpha}_{Ai}, \dot{\beta}_{Ai}$	inner and outer distribution rate; $i = 1, 2, \dots, n$; rad/s

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
$\dot{\alpha}_L, \dot{\beta}_L$	inner and outer rate limit; rad/s
$\dot{\alpha}_{LIM}, \dot{\beta}_{LIM}$	inner and outer rate limit due to hardware; rad/s
$\dot{\alpha}_{TLIM}, \dot{\beta}_{TLIM}$	inner and outer rate limit due to T_{LIM} ; rad/s
$\Delta\beta$	ideal outer gimbal angle separation; rad
$\dot{\underline{\beta}}_C$	outer rate command vector (due to T_{CM2}, T_{CM3}); rad/s
$\dot{\beta}_{Ci}$	components of $\dot{\underline{\beta}}_C$; $i = 1, 2, \dots, n$, rad/s
—	a bar under a quantity denotes a vector
$[]^T$	a superscript T denotes a transpose on a vector or a matrix
$[]^\dagger$	a superscript dagger denotes the pseudo-inverse of the matrix

STEERING LAW FOR PARALLEL MOUNTED DOUBLE-GIMBALED CONTROL MOMENT GYROS

i. INTRODUCTION

There are several reasons why some space vehicles require angular momentum exchange devices in addition to (or maybe instead of) reaction control systems: Fine pointing, long mission duration, and lack of contamination. Fine pointing requires a restoring torque which is continuously variable over several orders of magnitude; whereas, reaction jets are on-off devices. Long mission duration can be handled by momentum exchange devices since they use electrical energy which can be replenished by solar cells, but reaction jets use mass expulsion and the fuel consumption over long periods becomes prohibitive. Momentum exchange devices will not contaminate the immediate space vehicle environment, in contrast to reaction jets.

Three types of momentum exchange devices are commonly used: reaction wheels (RW's), single-gimbaled control moment gyros (SG CMG's), and double-gimbaled control moment gyros (DG CMG's). RW's achieve momentum exchange by accelerating (or decelerating) about a fixed axis. Therefore, they have one degree of freedom and their energy content varies greatly, requiring relatively large average power. Wheel imbalance can generate disturbances at all frequencies. The momentum steering laws are simple, therefore only satellites or smaller space vehicles use RW's.

SG CMG's have constant momentum magnitudes which can be positioned anywhere in planes perpendicular to the individual gimbal axes. For a system of SG CMG's, singularities (where no control torque can be generated in some direction) are a problem, dictating either a complicated steering law or the inefficient use of the angular momentum.

DG CMG's have two degrees of freedom each which allows maximum use of the constant momentum magnitudes and the singularities are easily avoided. Maximum momentum usage coupled with minimum software requirements also mandates unlimited angular freedom of the gimbal axes (no gimbal stops and the associated problems). When these DG CMG's are mounted with their outer

gimbal axes parallel and when the steering law¹ can accommodate any number of CMG's, a generally applicable momentum exchange system can be devised which always has a spherical momentum envelope. The fact that the number of DG CMG's is also freely selectable allows a few standard size CMG's to satisfy any angular momentum requirement; however, DG CMG's will be used mainly on large space vehicles (Skylab, Space Shuttle).

Figure 1 shows the inner and outer gimbal angles α_i and β_i of the i-th CMG. With this definition, the total angular momentum of n CMG's is:

$$\underline{H}_G = \begin{bmatrix} H_{G1} \\ H_{G2} \\ H_{G3} \end{bmatrix} = \begin{bmatrix} + \sum (H_i \sin \alpha_i) \\ - \sum (H_i \cos \alpha_i \sin \beta_i) \\ + \sum (H_i \cos \alpha_i \cos \beta_i) \end{bmatrix}, \quad (1)$$

where $i = 1, 2, \dots, n$ and H_i is the momentum magnitude of the i-th CMG. The CMG momentum change is

$$\dot{\underline{H}}_G = [A]\dot{\underline{\alpha}} + [B]\dot{\underline{\beta}}, \quad (2)$$

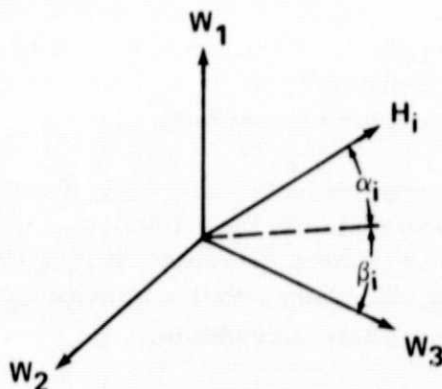


Figure 1. DG CMG gimbal angles.

1. The term steering law means here the set of equations necessary to generate a set of gimbal angle rate commands, which then result in the desired control torque on the space vehicle. This is achieved by steering the momentum vector of each CMG according to the commands generated by the steering law.

where

$$\dot{\underline{H}}_G = [\dot{H}_{G1} \dot{H}_{G2} \dot{H}_{G3}]^T, \quad (3)$$

$$[A] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ a_{31} & a_{32} & \dots & a_{3n} \end{bmatrix}, \quad (4)$$

$$[B] = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ b_{31} & b_{32} & \dots & b_{3n} \end{bmatrix}, \quad (5)$$

$$\dot{\underline{\alpha}} = [\dot{\alpha}_1 \dot{\alpha}_2 \dots \dot{\alpha}_n]^T, \quad (6)$$

$$\dot{\underline{\beta}} = [\dot{\beta}_1 \dot{\beta}_2 \dots \dot{\beta}_n]^T, \quad (7)$$

$$a_{1i} = +H_i c\alpha_i, \quad (8)$$

$$a_{2i} = +H_i s\alpha_i s\beta_i, \quad (9)$$

$$a_{3i} = -H_i s\alpha_i c\beta_i, \quad (10)$$

$$b_{1i} = 0, \quad (11)$$

$$b_{2i} = -H_i c\alpha_i c\beta_i, \quad (12)$$

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and

$$b_{3i} = -H_i \cos \alpha_i \sin \beta_i, \quad (13)$$

The torque exerted on the CMG system is

$$\underline{T}_G = \dot{\underline{H}}_G, \quad (14)$$

where

$$\underline{T}_G = [T_{G1} \ T_{G2} \ T_{G3}]^T$$

and it is convenient to define the control torque command \underline{T}_C as

$$\underline{T}_C = \dot{\underline{H}}_{GC}, \quad (15)$$

where

$$\underline{T}_C = [T_{C1} \ T_{C2} \ T_{C3}]^T \quad (16)$$

and

$$\dot{\underline{H}}_{GC} = [\dot{H}_{GC1} \ \dot{H}_{GC2} \ \dot{H}_{GC3}]^T \quad (17)$$

is the desired momentum change.

The mounting of all outer gimbal axes parallel to the same axis (discussed in detail in Section II) makes the control torque generated along this axis only a function of the inner gimbal angles and their rates (eqs. 1-13). This fact allows decomposition of the total three-dimensional control problem into a linear one for the inner rate commands and a planar one for the outer rate commands (Section IV). Since there are more than three degrees of freedom for the total system, an infinity of choices exists for the gimbal rate commands while satisfying the commanded control torque. The pseudo-inverse method is selected to give the basic set of gimbal rate commands. A two step approach is used. The inner rate commands are calculated first, and they are then considered known quantities in the calculation of the outer gimbal rate commands, both times using the pseudo-inverse method.

However, since the pseudo-inverse does not prevent singular states, a set of non-torque-producing inner and outer gimbal rate commands are added to the basic set (Section III), such that all singularities (besides saturation) are avoided.

Proportional gimbal rate command limiting is applied to all commands in the case any one of the gimbal rate commands exceeds the respective torquer capabilities. This ensures that only the magnitude of the generated torque is reduced, but the direction of the torque vector is not altered.

This report is the result of simplifications mentioned in Reference 1, but it was possible to carry the simplification so far as to eliminate the need for the variable weights of the pseudo-inverse which were used to invert the nonsquare $[A]$ and $[B]$ matrices. Therefore this report presents actually a different approach to the steering law problem.²

II. PARALLEL MOUNTING ARRANGEMENT

The proposed mounting, with all outer gimbal axes parallel, is shown in Figure 2. Although the outer gimbal axes are shown colinear, this is not a requirement. This mounting arrangement has many advantages. The mounting interfaces can be identical, i.e., mounting brackets and hardware, cable harnesses, etc. There is no need to individually identify the DG CMG's, and the onboard computer can assign an arbitrary label to any CMG, which could be different from one computation cycle to the next. This simplifies the steering law and the redundancy management. The parallel mounting of the outer gimbal axes in conjunction with a steering law that accepts any number of DG CMG's also makes failure accommodation a built-in feature. On the other hand, if

2. The steering law was called "control law" in Reference 1.

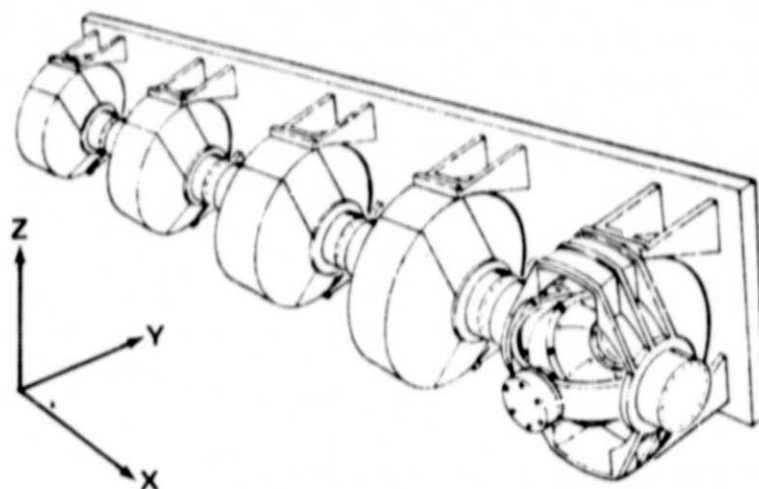


Figure 2. Parallel mounting arrangement.

increasing momentum requirements during the design of a vehicle (and the moments of inertia always tend to increase) demand it, an additional CMG can be added with minimum impact on hardware and software. The parallel mounting of DG CMG's also makes visualization of system operation exceedingly simple (especially when compared with the momentum envelope of skewed mounted single gimbal CMG's).

The only disadvantage of the parallel mounting lies in the fact that a large momentum demand along the direction of the outer gimbal axes reduces the maximum response in a plane perpendicular to the outer axes, since a small movement of the momentum vector demands a large outer gimbal rate. This possible problem can be eliminated by mounting the outer gimbal axes in the direction of the minimum momentum requirement (usually the minimum principal moment of inertia axis).

III. DESIRABLE GIMBAL ANGLE DISTRIBUTIONS

The n DG CMGs have $2n$ degrees of freedom. Three are needed to satisfy the torque command; the excess of $2n-3$ degrees of freedom are utilized to achieve a desirable gimbal angle distribution. Before one can decide on a desirable distribution, the characteristics of a DG CMG have to be considered.

The inner gimbal rate needed to produce a given torque perpendicular to the inner gimbal axis is independent of the inner and the outer gimbal angles. However, the outer gimbal rate needed to produce the same torque perpendicular to the outer gimbal axis is inversely proportional to the cosine of the inner

gimbal angle. Therefore it is desirable to keep the cosines of the inner gimbal angles high; i.e., it is desirable to minimize the maximum inner gimbal angle, which, in turn, reduces the outer rate requirements. The maximum inner gimbal angle is minimized when all inner gimbal angles are equal (also compare equation (1)) to the inner gimbal reference angle

$$\alpha_R = \sin^{-1} \left\{ \frac{\sum_i (H_i \sin \alpha_i)}{\sum_i H_i} \right\} \quad (18)$$

The simplicity of equation (18) is the direct result of the parallel mounting.

The situation is not as clear-cut with respect to the desirable distribution for the outer gimbal angles. However, for DG CMGs, a singular condition inside the total momentum envelope can only occur when some of the vectors (at least one) are antiparallel and the others parallel to the resultant. Maintaining adequate and more or less equal spacing between the vectors will eliminate the possibility of a singularity. In this report, a distribution is used which maximizes the minimum separation between any two outer gimbals. The angles $\Delta\beta$ between adjacent vectors are then equal, as shown in Figure 3 for three, four, and five DG CMGs (for two DG CMGs, the only excess degree of freedom is used for the equalization of the inner gimbal angles). Each of the vectors is halfway between its neighbors (the extreme vectors being obvious exceptions). The resultant momentum in the $V_2 - V_3$ plane then bisects the angle between the two extreme vectors for the case where all momentum magnitudes are equal and the inner gimbal angles are equal.

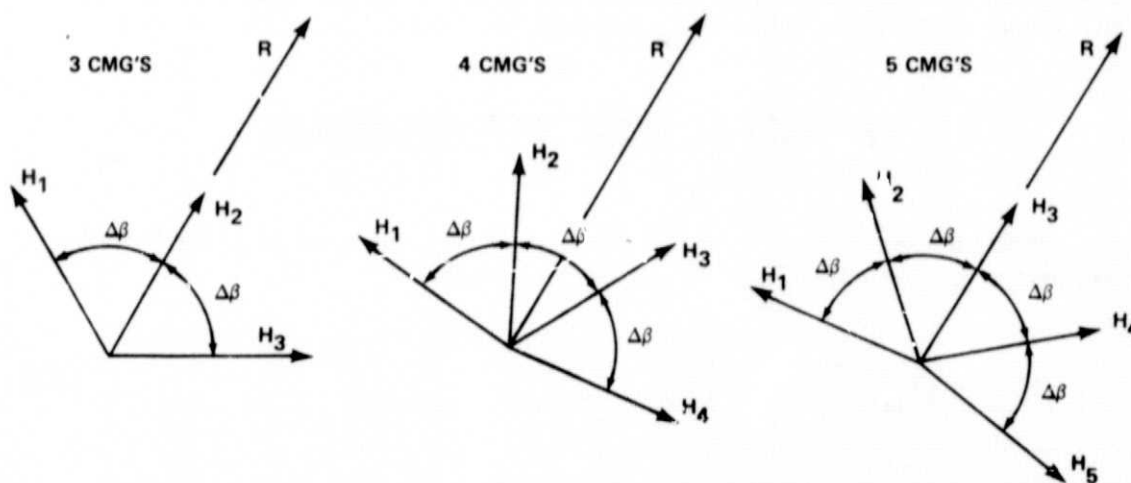


Figure 3. Outer gimbal angle distribution.

The calculation of the value for $\Delta\beta$ involves the solution of a transcendental equation, which in addition has a different form depending on whether the number of CMG's is odd or even. These difficulties can be avoided by the generation of a set of reference angles β_{Ri} which lie halfway between the adjacent vectors:

$$\beta_{Rk_1} = \left[\left(\beta_{k_n} + 2\pi \right) + \beta_{k_2} \right] / 2, \quad (19)$$

$$\beta_{Rk_i} = \left[\beta_{k_{i-1}} + \beta_{k_{i+1}} \right] / 2, \quad (20)$$

and

$$\beta_{Rk_n} = \left[\beta_{k_{n-1}} + \left(\beta_{k_1} - 2\pi \right) \right] / 2, \quad (21)$$

where k_1 is generated by an ordering scheme such that the value of k_1 is the index of the largest β_i , k_2 is the value of the next smaller one, etc. Details of the logic needed can be seen from the flowcharts in Appendix A. In the flowcharts, the possibility of one or more failed CMG's ($H_i = 0$) is also taken into account by replacing n by $n-m$, where now n is the original number of CMG's and m is the number of failed CMG's.

IV. INNER GIMBAL RATE COMMANDS

The change of the V_1 angular momentum component is not a function of the outer gimbal rates (see equations 1 through 14),

$$\dot{H}_{G1} = T_{G1} = \sum_i H_i c_{\alpha_i} \dot{\alpha}_i. \quad (22)$$

After a summation over j is done, an inner gimbal rate command, $\dot{\alpha}_{Ci}$, of the form

$$\dot{\alpha}_{Ci} = \left[H_i c\alpha_i / \sum_j (H_j c\alpha_j)^2 \right] T_{C1} \quad (23)$$

will result in $T_{G1} = T_{C1}$, if it is assumed that the actual and the commanded gimbal rates are equal.

After a summation over j is done, the torque equivalence can be shown by

$$\begin{aligned} T_{G1} &= \sum_j \left\{ H_i c\alpha_i \left[H_i c\alpha_i / \sum_j (H_j c\alpha_j)^2 \right] T_{C1} \right\} \\ &= \left[T_{C1} / \sum_j (H_j c\alpha_j)^2 \right] \sum_i H_i c\alpha_i H_i c\alpha_i \\ &= T_{C1} \left[\sum_i (H_i c\alpha_i)^2 / \sum_j (H_j c\alpha_j)^2 \right] \quad (24) \end{aligned}$$

The effect of the inner gimbal rates on the V_2 and V_3 momentum components is treated later.

Simultaneously with satisfying the torque command, it is desirable to reduce the maximum inner gimbal angle as discussed in the previous section. The inner gimbal angles can be moved toward equality without a V_1 torque being generated, as shown in Appendix B, by an inner gimbal rate addition, $\dot{\alpha}_{Ai}$, of the form

$$\dot{\alpha}_{Ai} = K_A (\alpha - \alpha_i) \quad (25)$$

where

$$\alpha = \sum_j \left(\alpha_j H_j c\alpha_j / \sum_j (H_j c\alpha_j)^2 \right) \quad (26)$$

and K_A is a constant gain ($1/K_A$ is the time constant). The angle α closely approximates the inner gimbal reference angle α_R , as shown in Appendix B.

The combined inner gimbal rate command is then³

$$\dot{\alpha}_i = \left[H_i c\alpha_i / \sum_j (H_j c\alpha_j)^2 \right] T_{C1} + K_A (\alpha - \alpha_i) \quad . \quad (27)$$

Rate limiting will be discussed after the outer gimbal rate commands have been established.

V. OUTER GIMBAL RATE COMMANDS

Again the total rate command is split into a rate addition for vector distribution and a rate to satisfy the torque command.

The preliminary outer gimbal rate additions are

$$\dot{\beta}'_{Ai} = K_A (\beta_{Ri} - \beta_i) \quad , \quad (28)$$

with the exception of the two outermost vectors which will have zero preliminary additional rate. The two outermost vectors are defined by the two smallest dot products between each of the vectors and the resultant in the V_2-V_3 plane. See the functions between points C and D on the flow chart in Appendix A for details. The sum of the $\dot{\beta}_{Ai}$ terms do not result in zero torque in the V_2-V_3 plane; in fact, the $\dot{\beta}$ terms required to compensate for the torque remove most of the $\dot{\beta}'_{Ai}$ and therefore almost defeat the distribution scheme. For redistribution, however, only relative movements are important and, after the summation over j is done, the final gimbal rate additions are

$$\dot{\beta}_{Ai} = \dot{\beta}'_{Ai} - \sum_j \dot{\beta}'_{Aj} \quad . \quad (29)$$

3. Throughout the report it is assumed that the final actual and the commanded gimbal rates are equal and both are called $\dot{\alpha}_i$ (or $\dot{\beta}_i$).

The disturbance torque caused by the $\dot{\beta}_{Ai}$ terms are

$$T'_{G2} = - \sum_i (H_i c\alpha_i c\beta_i \dot{\beta}_{Ai}) \quad (30)$$

and

$$T'_{G3} = - \sum_i (H_i c\alpha_i s\beta_i \dot{\beta}_{Ai}) \quad (31)$$

The inner gimbal rates also cause a disturbance torque in the $V_2 - V_3$ plane,

$$T''_{G2} = + \sum_i (H_i s\alpha_i s\beta_i \dot{\alpha}_i) \quad (32)$$

and

$$T''_{G3} = - \sum_i (H_i s\alpha_i c\beta_i \dot{\alpha}_i) \quad (33)$$

Only the remaining torque needs to be produced by the $\dot{\beta}_{Ci}$ terms:

$$T_{CM2} = T_{C2} - T'_{G2} - T''_{G2} \quad (34)$$

and

$$T_{CM3} = T_{C3} - T'_{G3} - T''_{G3} \quad (35)$$

or

$$T_{CM2} = T_{C2} + \sum [H_i (c\alpha_i c\beta_i \dot{\beta}_{Ai} - s\alpha_i s\beta_i \dot{\alpha}_i)] \quad (36)$$

and

$$T_{CM3} = T_{C3} + \sum [H_i (c\alpha_i s\beta_i \dot{\beta}_{Ai} + s\alpha_i c\beta_i \dot{\alpha}_i)] \quad (37)$$

We now apply the pseudo-inverse to get a unique solution,

$$\dot{\underline{\beta}}_C = [B']^\dagger \begin{bmatrix} T_{CM2} \\ T_{CM3} \end{bmatrix} \quad (38)$$

where

$$[B']^\dagger = [B']^T \left\{ [B'] [B']^T \right\}^{-1} \quad (39)$$

and

$$[B'] = \begin{bmatrix} b_{21} & b_{22} & \dots & b_{2n} \\ b_{31} & b_{32} & \dots & b_{3n} \end{bmatrix} \quad (40)$$

The total outer gimbal rate commands are then

$$\dot{\beta}_i = \dot{\beta}_{Ai} + \dot{\beta}_{Ci} \quad (41)$$

VI. PROPORTIONAL GIMBAL RATE LIMITING

The gimbal torquers will have a definite torque limit, T_{LIM} . In order not to exceed this limit, we have to establish the torque demand on the torquers due to the gimbal rates. The CMG torque is therefore resolved into the outer gimbal coordinate system (the W_1 axis is parallel to the outer gimbal axis, the W_2 axis is parallel to the inner gimbal axis),

$$\begin{bmatrix} T_{1i} \\ T_{2i} \\ T_{3i} \end{bmatrix} = H_i \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\beta_i & s\beta_i \\ 0 & -s\beta_i & c\beta_i \end{bmatrix} \begin{bmatrix} \dot{\alpha}_i c\alpha_i \\ \dot{\alpha}_i s\alpha_i s\beta_i - \dot{\beta}_i c\alpha_i c\beta_i \\ -\dot{\alpha}_i s\alpha_i c\beta_i - \dot{\beta}_i c\alpha_i s\beta_i \end{bmatrix}$$

$$= H_i \begin{bmatrix} +\dot{\alpha}_i c\alpha_i \\ -\dot{\beta}_i c\alpha_i \\ -\dot{\alpha}_i s\alpha_i \end{bmatrix}, \quad (42)$$

where now T_{1i} is the outer gimbal torque (after passing through the inner gimbal bearing), T_{2i} is the inner gimbal torque (which then passes through the outer gimbal bearing), and T_{3i} is carried by both the inner and outer gimbal bearings.

Assuming the same torque limit for the inner and outer gimbal torquers, we get for the gimbal rate limits due to the torque limit,

$$\dot{\alpha}_{TLIM} = \dot{\beta}_{TLIM} = T_{LIM} / (H_i c\alpha_i) \quad (43)$$

There are also fixed rate limits ($\dot{\alpha}_{LIM}, \dot{\beta}_{LIM}$) due to other hardware limits (gimbal rate tachometer limit, voltage limits, etc.).

To reduce the magnitude of the actual torque only, but keep the same direction as the commanded torque, a proportional scaling of all gimbal rates by dividing by D is done, where:

$$D = \text{MAX} \left\{ 1, \frac{|\dot{\alpha}_1|}{\dot{\alpha}_L}, \frac{|\dot{\alpha}_2|}{\dot{\alpha}_L}, \dots, \frac{|\dot{\alpha}_n|}{\dot{\alpha}_L}, \right. \\ \left. \frac{|\dot{\beta}_1|}{\dot{\beta}_L}, \frac{|\dot{\beta}_2|}{\dot{\beta}_L}, \dots, \frac{|\dot{\beta}_n|}{\dot{\beta}_L} \right\}, \quad (44)$$

with

$$\dot{\alpha}_L = \text{MIN} (\dot{\alpha}_{TLIM}, \dot{\alpha}_{LIM}) \quad (45)$$

and

$$\dot{\beta}_L = \text{MIN} (\dot{\beta}_{TLIM}, \dot{\beta}_{LIM}) \quad (46)$$

VII. PERFORMANCE DISCUSSION

The steering law performance was checked out and verified with a hybrid computer simulation. No logic flaws or unanticipated behavior was found.

As implied by the discussion on the desirable gimbal angle distribution, there is no need for a strict adherence to the ideal distribution. For the outer gimbal angles, this means that the time constant $(1/K_A)$ can be chosen such that, while a close adherence for a large V_2 - V_3 momentum resultant is achieved, small circles of the V_2 - V_3 resultant about the V_1 axis do not result in relatively large distributive outer rates.

The difference between an outer gimbal angle and its reference cannot exceed $\pi/2$. The maximum distribution rate β_{Ai} is then $K_A \pi/2$, as shown in equations (28) and (29).

A more involved gimbal rate limiting procedure can be implemented if the distribution rates tend to interfere with the control torque generation. Details are shown in Appendix B.

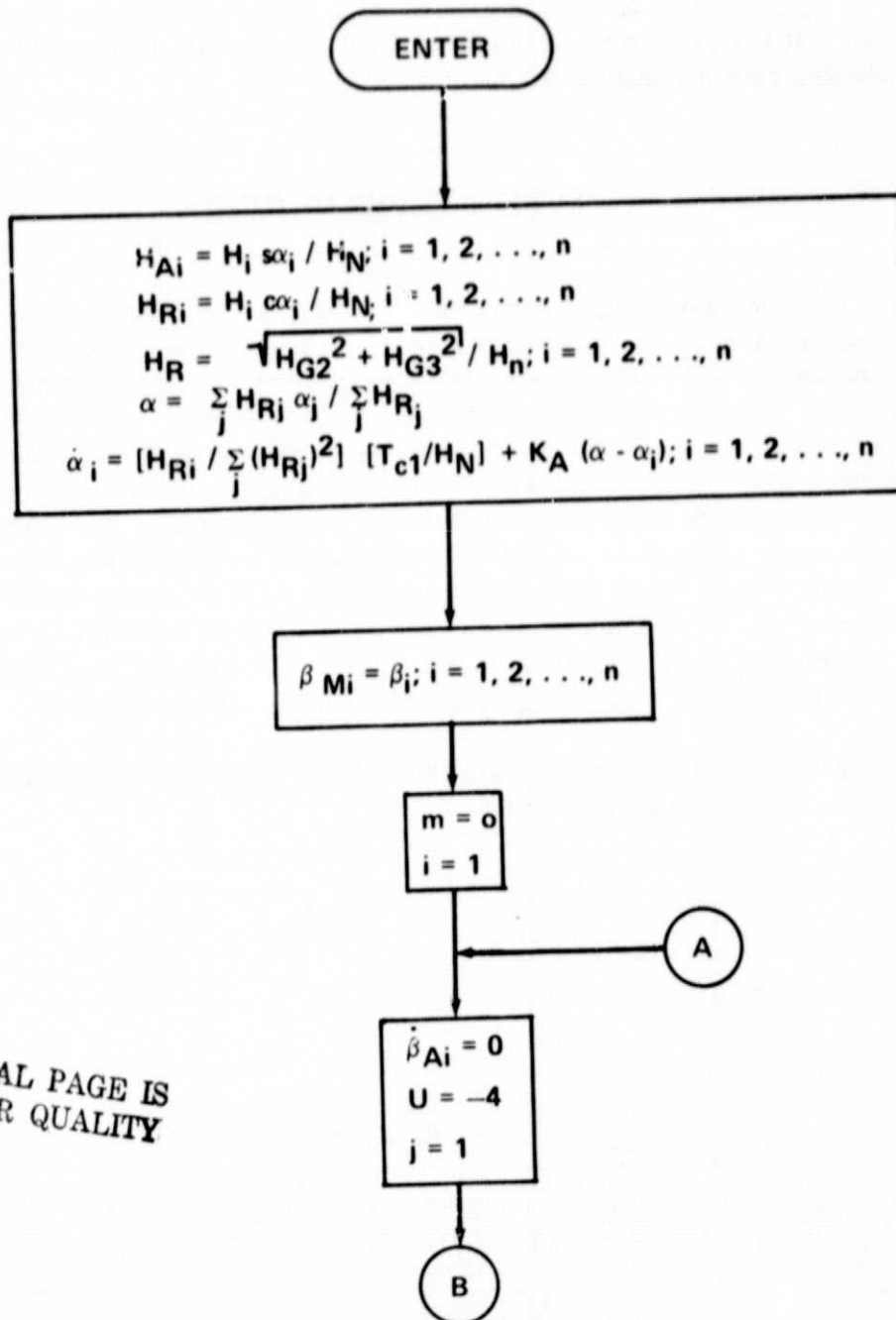
VIII. CONCLUSIONS

Parallel mounting of stopless DG CMG's allows the angular momentum steering problem to be split into a linear and a planar part which simplifies the steering law. As a consequence, the steering law can accommodate any number of DG CMG's within reasonable software requirements. The fact that any number of CMG's can be accommodated makes treatment of CMG failures a built-in property. The capability for clustering any number of CMG's allows the satisfying of any system angular momentum requirement with a few standard sizes. The maximum number of DG CMG's necessary in a cluster would be five if the next larger size is twice the smaller one (assuming — before failure — no less than three CMG's are considered). The steering-law/parallel-mounted-DG CMG combination can therefore be considered a "CMG KIT" applicable to any space vehicle where the need for DG CMG's has been established.

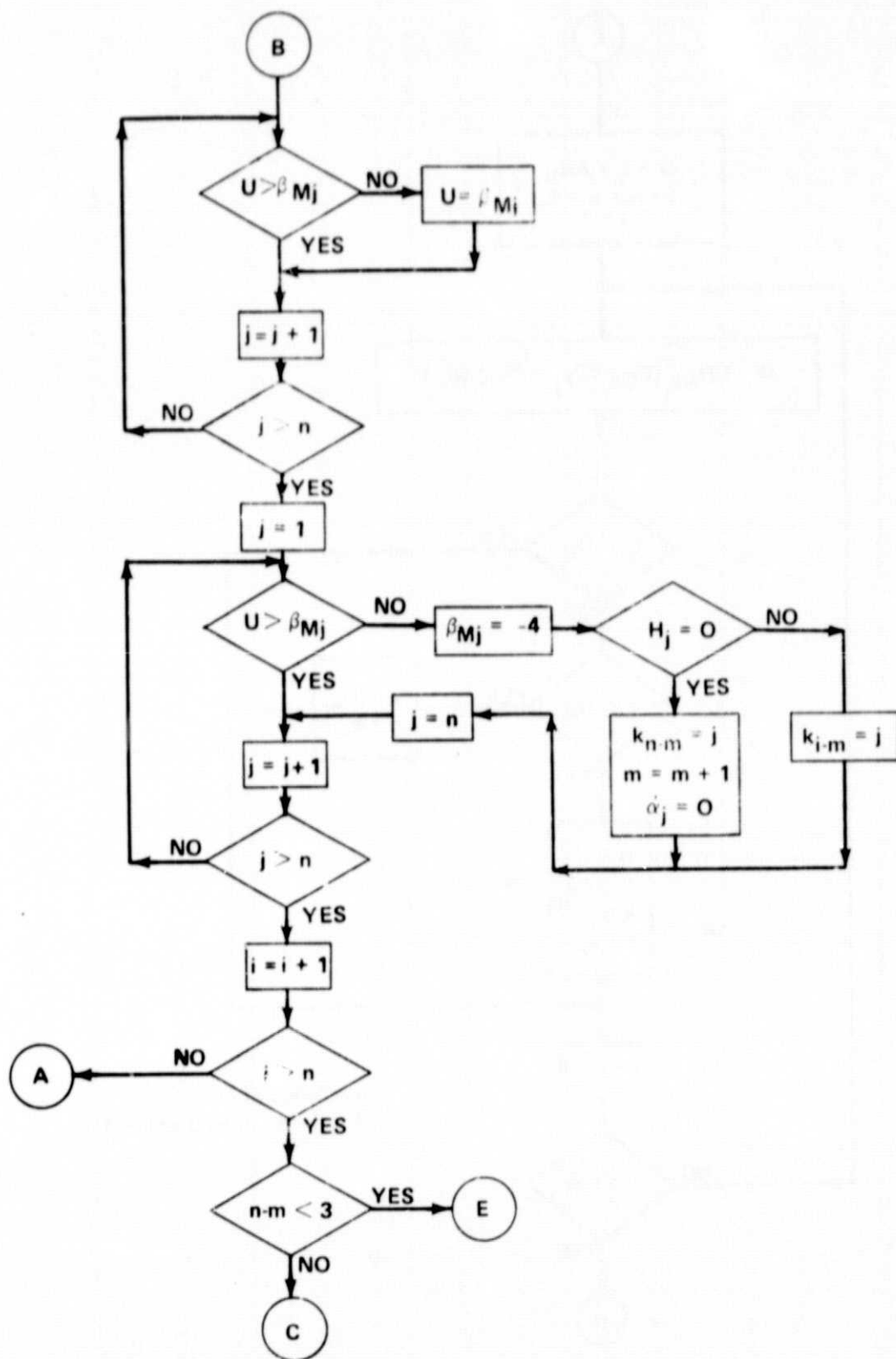
The logic flow charts of Appendix A are being redesigned and a 30 per cent reduction in logic is anticipated.

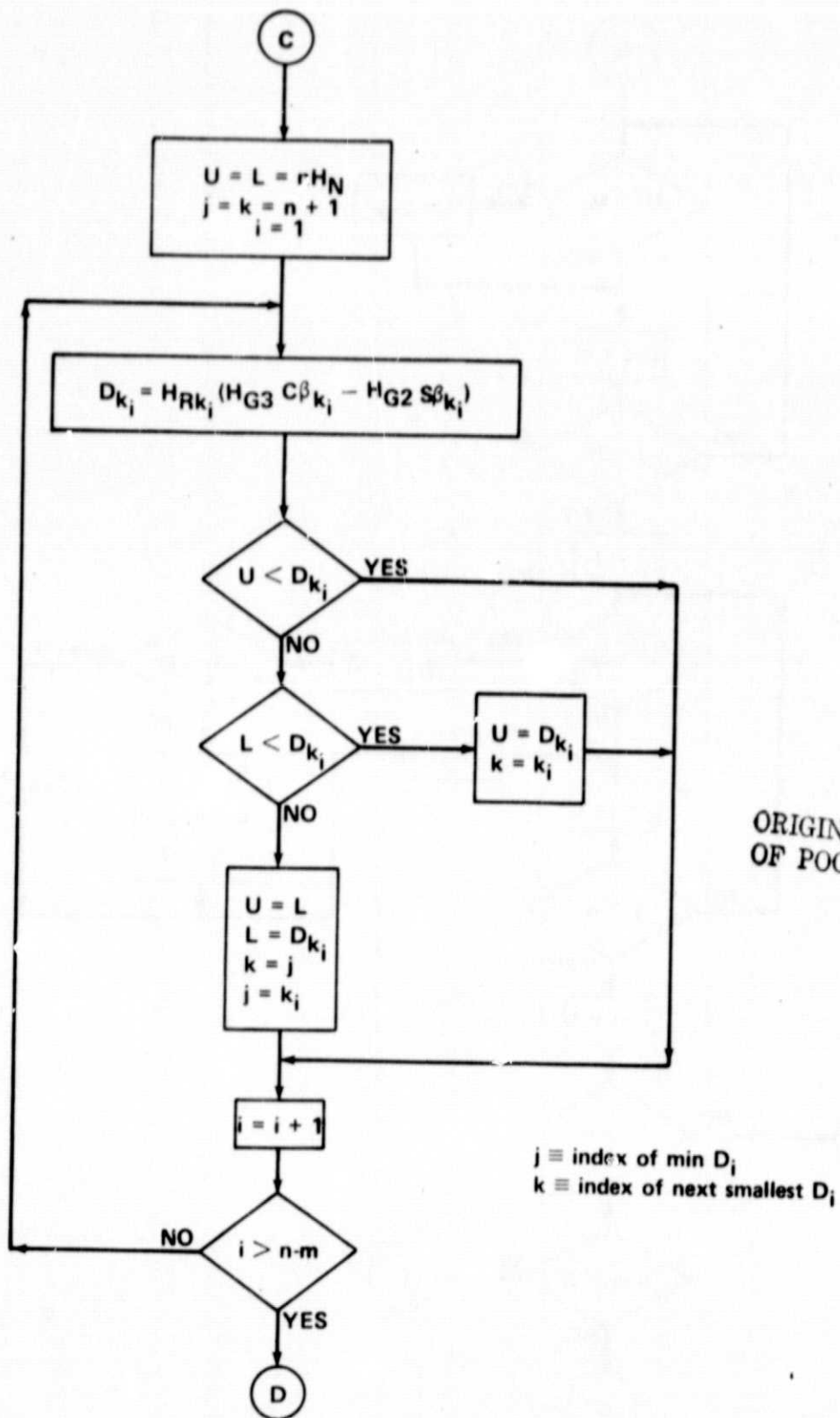
APPENDIX A

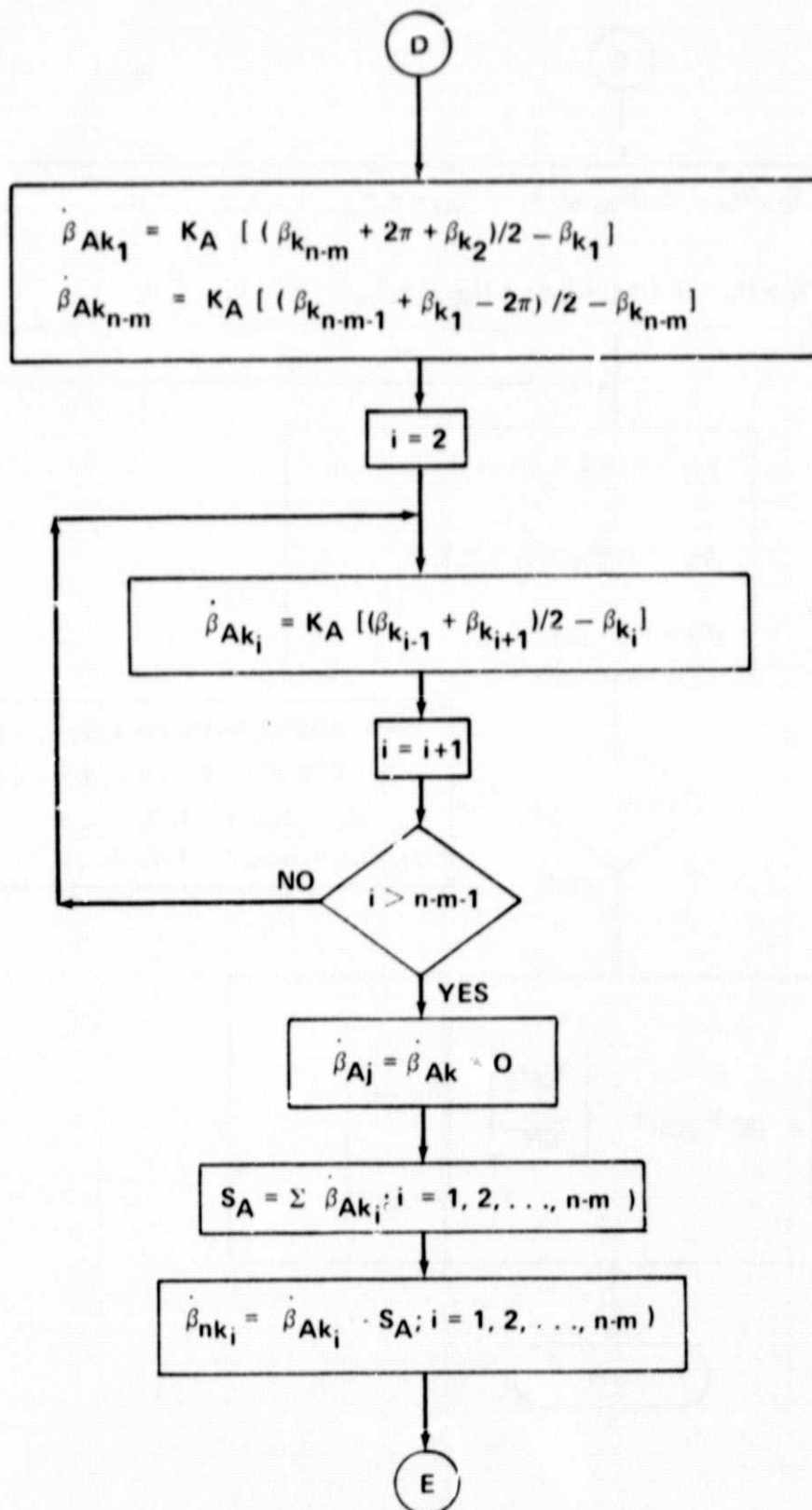
STEERING LAW LOGIC FLOW CHARTS

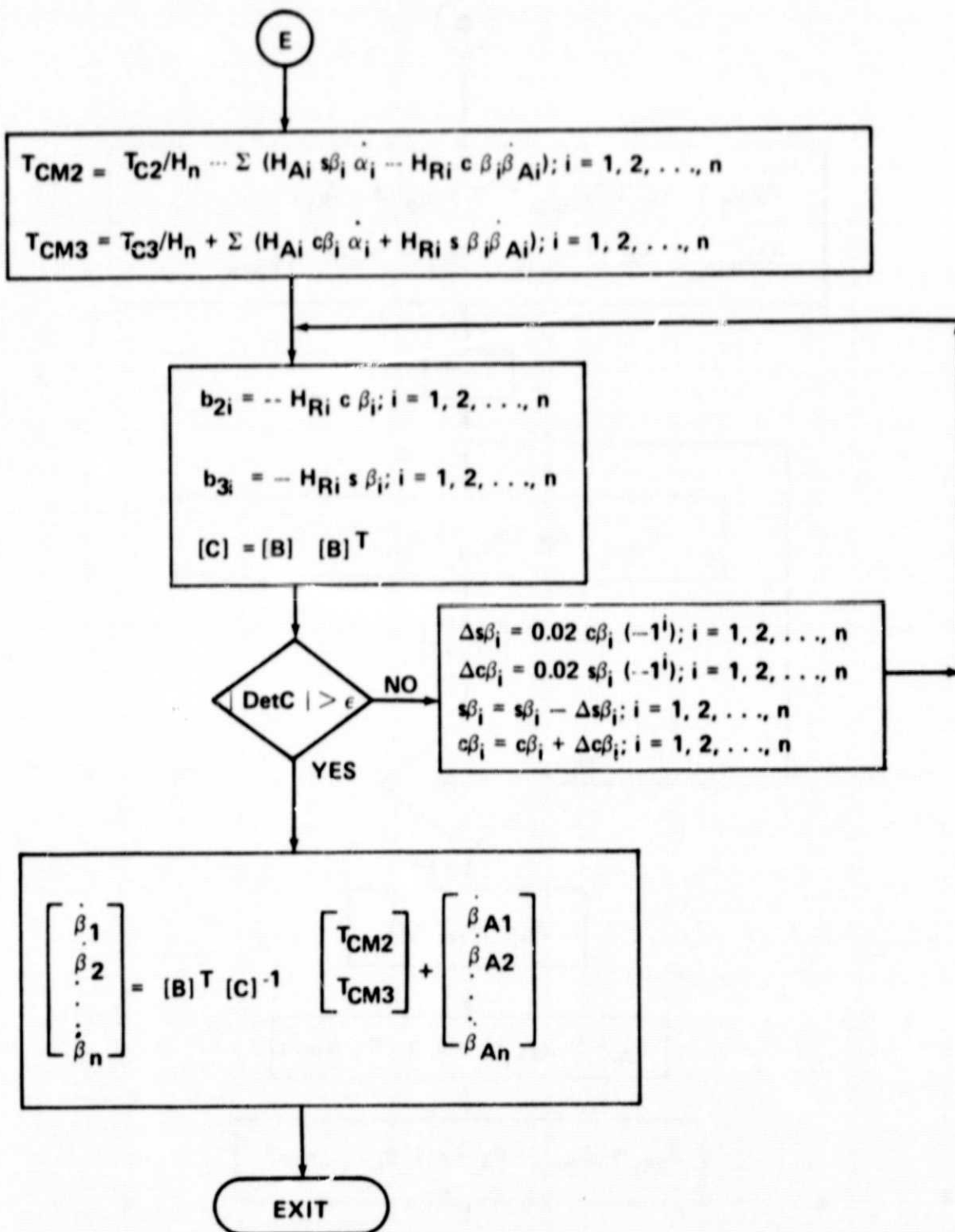


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APPENDIX B

SUPPLEMENTAL CALCULATIONS

1. Zero Torque Disturbance on V_1 -Component. After the summations over j were done, the inner gimbal rate addition (for distribution) was shown in equations (25) and (26) as

$$\dot{\alpha}_{Ai} = K_A (\alpha - \alpha_i) \quad ,$$

where

$$\alpha = \frac{\sum_j (H_j c_{\alpha j} \alpha_j)}{\sum_j (H_j c_{\alpha j})} \quad .$$

The V_1 -torque due to $\dot{\alpha}_{Ai}$ terms is

$$\begin{aligned} T_{A1} &= \sum_i (H_i c_{\alpha i} \dot{\alpha}_{Ai}) \\ &= \sum_i \left(H_i c_{\alpha i} \left\{ K_A \left[\frac{\sum_j (H_j c_{\alpha j} \alpha_j)}{\sum_j (H_j c_{\alpha j})} - \alpha_i \right] \right\} \right) \\ &= K_A \left\{ \left[\frac{\sum_j (H_j c_{\alpha j} \alpha_j)}{\sum_j (H_j c_{\alpha j})} \right] \sum_i (H_i c_{\alpha i}) - \sum_i (H_i c_{\alpha i} \alpha_i) \right\} \\ &= K_A \left\{ \sum_j (H_j c_{\alpha j} \alpha_j) - \sum_i (H_i c_{\alpha i} \alpha_i) \right\} \\ &= 0 \quad , \end{aligned} \tag{B-1}$$

i.e., no V_1 torque is generated for any combination of H_i and α_i values.

2. Convergence of α to α_R . The angle α closely approximates the inner reference angle α_R of equation (18). This can be shown by assuming small deviations $\Delta\alpha_i$ from α_R :

$$\alpha_i = \alpha_R + \Delta\alpha_i \quad (\text{B-2})$$

and

$$\sin \alpha_R = \frac{\sum_i [H_i \sin(\alpha_R + \Delta\alpha_i)]}{\sum_i H_i} \quad (\text{B-3})$$

For $\Delta\alpha_i \ll 1$, we get

$$\sin \alpha_R \sum_i H_i = \sin \alpha_R \sum_i H_i + \cos \alpha_R \sum_i (H_i \Delta\alpha_i) \quad (\text{B-4})$$

or

$$\sum_i (H_i \Delta\alpha_i) = 0 \quad (\text{B-5})$$

The angle α becomes

$$\alpha = \frac{\sum_i [H_i \cos(\alpha_R + \Delta\alpha_i)(\alpha_R + \Delta\alpha_i)]}{\sum_i [H_i \cos(\alpha_R + \Delta\alpha_i)]} \quad (\text{B-6})$$

or

$$\begin{aligned} \alpha \left[\cos \alpha_R \sum_i H_i - \sin \alpha_R \sum_i (H_i \Delta\alpha_i) \right] &= \alpha_R \left[\cos \alpha_R \sum_i H_i - \sin \alpha_R \sum_i (H_i \Delta\alpha_i) \right] \\ &\quad + \cos \alpha_R \sum_i (H_i \Delta\alpha_i) \quad (\text{B-7}) \end{aligned}$$

For $\Delta\alpha \ll 1$, we get

$$\alpha - \alpha_R = \left[\sum_i (H_i \Delta\alpha_i) / \sum_i H_i \right] / \left\{ 1 - \tan \alpha_R \left[\sum_i (H_i \Delta\alpha_i) / \sum_i H_i \right] \right\} \quad (B-8)$$

Since $\sum_i H_i \Delta\alpha_i = 0$, the difference between α and α_R is also zero.

3. Expanded Gimbal Rate Limiting. If the torque demands are large when compared with the gimbal rate limits, a more involved limiting procedure can be implemented. When the system is pushed, the outer gimbal rates due to the distribution could subtract from the ones needed for the torque generation. The inner distribution rates are always small - after the initial transient has died out - since the torque generation itself tends to keep the inner gimbal angles equal and therefore on the distribution.

The inner rates are calculated as shown by equation (27) and then divided by

$$D_\alpha = \text{MAX} (1, |\dot{\alpha}_1|/\dot{\alpha}_L, |\dot{\alpha}_2|/\dot{\alpha}_L, \dots, |\dot{\alpha}_n|/\dot{\alpha}_L),$$

where

$$\dot{\alpha} = \text{MIN} (\dot{\alpha}_{TLIM}, \dot{\alpha}_{LIM}) \quad (B-9)$$

This effects limiting, if necessary.

The outer gimbal rates for the control torque generation are calculated next, using equation (38),

$$\dot{\beta}_C = [B']^\dagger \begin{bmatrix} T_{C2} - T''_{G2} \\ T_{C3} - T''_{G3} \end{bmatrix},$$

and then divided by

$$D = \text{MAX} (1, D_{\alpha}, D_{\beta}) \quad , \quad (\text{B-10})$$

where

$$D_{\beta} = \text{MAX} (|\dot{\beta}_{C1}|/\dot{\beta}_L, |\dot{\beta}_{C2}|/\dot{\beta}_L, \dots, |\dot{\beta}_{Cn}|/\dot{\beta}_L) \quad (\text{B-11})$$

and

$$\dot{\beta}_L = \text{MIN} (\dot{\beta}_{TLIM}, \dot{\beta}_{LIM}) \quad . \quad (\text{B-12})$$

This effects limiting, if necessary. Next, the outer distribution rates are calculated (equation 29) with

$$\dot{\underline{\beta}}'' = \dot{\underline{\beta}}'_{Ai} - \sum_j \dot{\beta}'_{Aj}$$

To eliminate a torque disturbance, these rates have to be further modified to

$$\dot{\underline{\beta}}_A = \{E - [B']^{\dagger} [B']\} \dot{\underline{\beta}}''_A \quad , \quad (\text{B-13})$$

where E is the $n \times n$ identity matrix. The $\dot{\beta}_{Ai}$ are then divided by

$$D_A = \text{MAX} (1, |\dot{\beta}_{A1}|/\dot{\beta}_{AL}, |\dot{\beta}_{A2}|/\dot{\beta}_{AL}, \dots, |\dot{\beta}_{An}|/\dot{\beta}_{AL}) \quad , \quad (\text{B-14})$$

where

$$\dot{\beta}_{AL} = \dot{\beta}_L (1 - D_{\beta}/D_{\alpha}) \quad (B-15)$$

is the left over capability after the torque generation demand is met. If $\dot{\beta}_{AL}$ is less or equal to zero, all $\dot{\beta}_{Ai}$ are set to zero.

REFERENCE

1. Kennel, H. F.: Control Law for Parallel Mounted Double-Gimbaled Control Moment Gyros. Proceedings of the Second Annual Research and Technology Review, October, 1974, Marshall Space Flight Center.


APPROVAL

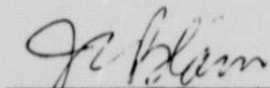
STEERING LAW FOR PARALLEL MOUNTED DOUBLE-GIMBALED CONTROL MOMENT GYROS

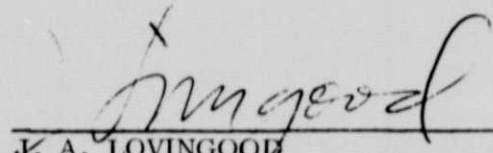
By H. F. Kennel

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.


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